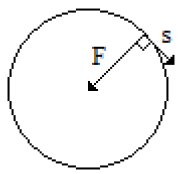
Work and Kinetic Energy

Notes prepared by Megan L. Barry

Based on Physics, 5th Ed. by Resnick, Halliday, Krane (Ch. 11)

1 Work Done by a Constant Force

- The work done by a constant force $\vec{F}$ that moves a body through a displacement $\vec{s}$ is given by $W = \vec{F} \cdot \vec{s} = Fs\cos\phi$, where ϕ is the angle between $\vec{F}$ and $\vec{s}$.
- Work must be calculated separately for each force on an object.
- Work is negative when the force acts opposite to the displacement direction (because $\cos 180 = -1$)
- Centripetal force does no work (because $\vec{s}$ is perpendicular to $\vec{F}$)



- Unit of work: Joule = N·m
  - 1 Joule is the work done by a force that exerts 1 N for 1 m
- Work depends on the inertial frame of the observer. Observers in other frames will measure the same force, but a different displacement.
- Power - the rate at which work is done
  - Unit: Watt = J/s

2 Power

- Average power $P_{av} = \frac{W}{t}$
- Instantaneous power $P = \frac{dW}{dt}$
- Work can be expressed as power*time (i.e. kilowatt-hours). 1 kW·h is the work done in 1 hour by an agent working at a constant rate of 1 kW.
- Power can also be expressed as $P = \vec{F} \cdot \vec{v}$ (because $P = \frac{dW}{dt} = \frac{\vec{F}\cdot\vec{s}}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt}$)
- Delivering negative power to a body means doing negative work on it

3 Work done by a variable force

- For a force that varies with position, the area under a plot of $F(x)$ gives the work done over that distance. $W = \int_{x_i}^{x_f} F_x(x)dx$
- Restoring force - a force that acts to restore a body to its location at x=0
  - Example: spring force $F(x) = -kx$
    - The spring constant k has units of N/m
    - A higher spring constant means a stiffer spring
  - In force-balancing diagrams, the sign of the spring force is determined by the current direction of the force
- Work done by a spring is given by $W_s = \frac{1}{2}kx^2$
  - Can be positive or negative depending on direction of force and displacement
- To calculate work done in 2 dimensions, you can use a line integral

4 Kinetic Energy and the Work-Energy Theorem

- Kinetic energy K of a body mass m moving at speed v is $K = \frac{1}{2}mv^2$
  - Unit: Joule
  - K is always positive
- Work-Energy theorem: The net work done by the forces acting on a body is equal to the change in kinetic energy of the body. That is, $W_{net} = \Delta K$
- Work is a way of transferring energy to or from a body via a force that acts on it
- Work is only done when the magnitude of the velocity changes (changing direction does no work)
- Two observers in different inertial reference frames may measure different values for W_{net} and ΔK, but they both find that $W_{net} = \Delta K$.
- Invariant - describes a law that has the same form in all inertial reference frames (such as work-energy theorem)
- Remember: $W_{net} = \Delta K$ only works for particles.

5 Work and Kinetic Energy in Rotational Motion

- The work done by a torque that rotates an object is given by $W = \int_{\theta_i}^{\theta_f} \tau_z d\theta$. For constant torque, $W = \tau_z \theta$

  - Power expended is given by $P = \tau_z \omega_z$ (from $P = \frac{d\omega}{dt} = \tau_z \frac{d\theta}{dt} = \tau_z \omega_z$)
  - Remember: θ must be in radians.

- Since $v = r\omega$, rotational kinetic energy is given by $K = \frac{1}{2} I \omega^2$.
- $W_{net} = \Delta K$ must account for work done by forces ¡u¿and¡/u¿ torques, and ΔK must account for translational ¡u¿and¡/u¿ rotational velocity.

6 Kinetic Energy in Collisions

- In an elastic collision, the total kinetic energy of the bodies remains constant. That is, $K_i = K_f$ (in *all* inertial frames)
  - In the CM frame, the kinetic energy of each individual object is also constant.
- In an inelastic collision, the total final kinetic energy is less than the total initial kinetic energy. That is, $K_f < K_i$. However, the total linear momentum is still constant.
- In a completely inelastic collision, the two bodies stick together, and lose the maximum amount of kinetic energy.
- In an explosive collision, the total final kinetic energy is greater than the total initial kinetic energy. That is, $K_f > K_i$.