Rotational Dynamics

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Based on Physics, 5th Ed. by Resnick, Halliday, Krane (Ch. 9)

1 Torque

- Torque - the source of an object's angular acceleration (just as force is the source of an object's translational acceleration). Depends on the applied force, as well as where on the object it was applied.
- Moment of inertia (AKA rotational inertia) - a scalar quantity that describes the distribution of an object's mass relative to its axis of rotation
  - Analogy: mass represents an object's resistance to linear acceleration, moment of inertia represents an object's resistance to angular acceleration
- Magnitude of torque is given by $\tau = rF\sin\theta$, where θ is the angle between $\vec{r}$ and $\vec{F}$ (τ is maximum when $\theta = 90$, and 0 when $\theta = 0$ or 180), where $\vec{r}$ is a vector from the origin to the point at which $\vec{F}$ is applied.
- Unit of torque: N·m "Newton-meters"
- The vector form of torque is given by $\vec{\tau} = \vec{r} \times \vec{F}$

2 Rotational Inertia and Newton's Second Law

- For a single particle, the moment of inertia is given by $I = mr^2$
  - From $\sum F_{Tangential} = ma_C => F\sin\theta = m\alpha_z r => \tau_z = mr^2\alpha_z = I\alpha_z$ (analogous to F=ma)
- Unit of moment of inertia: kg·m^2
- For n particles rotating about the same axis, $I = \sum m_n r_n^2$
- This leads to the rotational form of Newton's second law: the net external torque about a fixed axis is equal to the product of the total moment of inertia and the angular acceleration. That is, $\sum \tau_{ext,z} = I\alpha_z$
  - This formula cannot simply be written as a vector because I is different for each axis. I in its complete form is actually a tensor quantity.

- Parallel axis theorem: The moment of inertial of an object about an arbitrary axis is given by $I = I_{CM} + Mh^2$, where I_{CM} is the moment of inertia about an axis parallel to the chosen axis and passing through the center of mass, M is the object's mass, and h is the distance between the two axes.
  - Corollary: Mh^2 is always positive, so I_{CM} is the smallest moment of inertia for a set of parallel axis, thus choosing I_{CM} for a given torque maximizes α_z.

3 Rotational Inertia of Solid Bodies

- The moment of inertia of any solid object is given by $I = \int r^2 dm$
  - Use substitution $dm = \rho dV$

4 Torque due to Gravity

- The torque due to gravity on a solid object is given by $\tau_g = \vec{r}_{CM} \times M\vec{g}$
  - Thus, the torque due to gravity about an object's center of mass is zero.
- In many cases, an object's center of mass and center of gravity are the same. They are different if the value of the acceleration due to gravity $\vec{g}$ is significantly different for different points on the object.
- An object is said to be in mechanical equilibrium if the net force and net torque on it are zero.

5 Applications of Newton's Laws for Rotation

- For an object in translational equilibrium, the torque about any point is the same.
- In force-balancing equations, positive torque means that the force producing it would cause a counterclockwise rotation if acting alone.
- In general, translational and rotational motion can be considered independently.

6 Combined Rotational and Translational Motion

- When a circular object "rolls without slipping", there is no relative motion between the point of contact and the surface (the translational velocity cancels with the tangential velocity at that point, i.e. $v_{CM} = \omega R$)
  - Thus, μ_s would be used instead of μ_k when considering frictional forces.