Rotational Kinematics

Notes prepared by Megan L. Barry

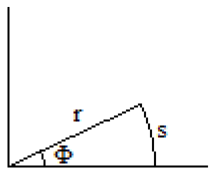
Based on Physics, 5th Ed. by Resnick, Halliday, Krane (Ch. 8)

1 Rotational Motion

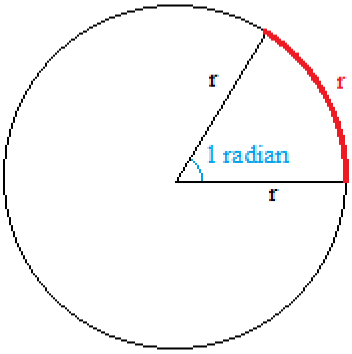
- Rotational kinematics - describing an object's rotation with appropriate variables and relating them to one another
- Rotational dynamics - relating rotational motion to the interaction of an object with its environment
- A rigid body moves in pure rotation (no translational motion) if every point in the body moves in a circular path centered on the axis of rotation (or, equivalently, if a reference line perpendicular to the axis moves through the same angle in a given time as any other similar reference line)
- 3D motion of a rigid body requires 6 coordinates to describe: 3 to locate the center of mass, 2 angles to orient the axis of rotation, and 1 angle to describe rotation about the axis.

2 The Rotational Variables

- Arc length $s = r\phi$, where ϕ is in radians



- Radian - an angle corresponding to an arc length equal to the radius of a circle (≈ 57.3)
  - $2\ \pi$ radians = 360 = 1 revolution



- For equations that mix linear and angular quantities, all angles must be in radians
- Angular displacement $\Delta\phi = \phi_f - \phi_i$
- Average angular velocity $\omega_{av} = \frac{\Delta\phi}{\Delta t}$
- Instantaneous angular velocity $\omega = \frac{d\phi}{dt}$
  - Positive for increasing ϕ, negative for decreasing ϕ
  - Dimensions: 1/seconds (can be rev/s or rad/s)
- For a rigid body in pure rotation, ω is the same at every point.
- Average angular acceleration $\alpha = \frac{\Delta\omega}{\Delta t}$
- Instantaneous angular acceleration $\alpha = \frac{d\omega}{dt}$
  - Dimensions: $\frac{1}{s^2}$ (can be rev/s^2 or rad/s^2)
  - Also the same throughout a rigid body in pure rotation

3 Rotational Quantities as Vectors

- Curl the fingers of your right hand in the direction of an object's rotation. Your thumb points in the direction of the angular velocity vector $\vec{\omega}$
- Even though ω seems to be proportional to scalars (ϕ and t), ω can be represented as a vector because very small angles can in fact be represented as vectors (i.e. $\frac{d\vec{\phi}}{dt} = \vec{\omega}$)

4 Rotation with Constant Angular Acceleration

- Integrating $a_z = \frac{d\omega_z}{dt}$ gives $\omega_z = \omega_{0,z} + \alpha_z t$
- Integrating $\omega_z = \frac{d\phi}{dt}$ gives $\phi = \phi_0 + \omega_{0,z}t + \frac{1}{2}\alpha_z t^2$
- ω_z is positive if ϕ is increasing. α_z is positive if ω_z is increasing.

5 Relationships between Linear and Angular Variables

- Tangential (linear) speed $v_T = \omega r$ (from taking the time derivative of $s - \phi r$)
- Tangential (linear) acceleration $a_T = \alpha r$ (from $\frac{dv_T}{dt}$)
- Combining this with $a_C = \frac{v^2}{r}$ gives $a_C = \omega^2 r$

6 Vector Relationships between Linear and Angular Variable

- Unit vectors for 2D rotation:
  - $\hat{\vec{r}}$ - points in direction of increasing r (outward)
  - $\hat{\vec{\phi}}$ - points in direction of increasing ϕ (tangential to circular path)
- In terms of $\hat{\vec{i}}$ and $\hat{\vec{j}}$,
  - $\hat{\vec{r}} = (\cos\phi)\hat{\vec{i}} + (\sin\phi)\hat{\vec{j}}$
  - $\hat{\vec{\phi}} = (-\sin\phi)\hat{\vec{i}} + (\cos\phi)\hat{\vec{j}}$
- In vector form,
  - $\vec{v} = v_T\hat{\vec{\phi}}$
  - $\vec{a} = \vec{a}_T + \vec{a}_C = a_T\hat{\vec{\phi}} - v_T\omega\hat{\vec{r}}$
- The direct relation between $\vec{v}$ and $\vec{\omega}$ is given by $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{r}$ is the object's position vector
- Taking $\frac{d\vec{v}}{dt}$ gives $\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$, where $\vec{a}_T = \vec{\alpha} \times \vec{r}$ and $\vec{a}_C = \vec{\omega} \times \vec{v}$