Fluid Dynamics

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Based on Physics, 5th Ed. by Resnick, Halliday, Krane (Ch. 16)

1 General Concepts of Fluid Flow

- For now, we will ignore viscous forces (dissipative forces in fluids)
- The general method we will use to analyze fluid flow involves specifying the density $\rho(x, y, z, t)$ and velocity $\vec{v}(x, y, z, t)$ of a fluid at point (x, y, z) and time t.
- Fluid flow can be steady or nonsteady. In steady fluid flow, the pressure, density, and velocity at each point is constant with time (they can still change from point to point)
  - Turbulent - describes fluid flow in which velocity varies erratically with time and position
- Fluid flow can be compressible or incompressible. Incompressible flow has constant density that is independent of position and time.
  - Of course liquids can be considered incompressible, but even air can be considered incompressible in some situations, such as in subsonic aerodynamics
- Fluid flow can be viscous or nonviscous. Viscosity in fluids can be thought of as being analogous to friction in solids.
- Fluid flow can be rotational or irrotational. In irrotational fluid flow, a particle flowing in the stream does not rotate about an axis.
- Ideal fluid - a fluid that is steady, incompressible, nonviscous, and irrotational

2 Streamlines and the Equation of Continuity

- Streamline - a path followed by every particle that passes through a given point in *steady* fluid flow. Since any particle passing through a given point has the same velocity, a particle passing through a certain point must follow the same path as any other particle that passes through that point.

- In general, the magnitude of velocity is not constant throughout a streamline.
- The direction of the velocity at any point is tangent to the streamline.
- Streamlines cannot cross one another (otherwise, a single point would have two $\vec{v}$ values).

- We can consider a bundle of streamlines, known as a"tube of flow". No streamline crosses the boundary of the tube. It must be narrow enough such that the flow rate v is constant over any cross-sectional area.

- Mass flux - the small amount of mass passing through tube cross section A in a short time. Given by $\frac{\delta m}{\delta t} = \rho A v$, where ρ and v are the fluid density and velocity at the cross section.
  - From $m = \rho V = \rho A v \delta t$

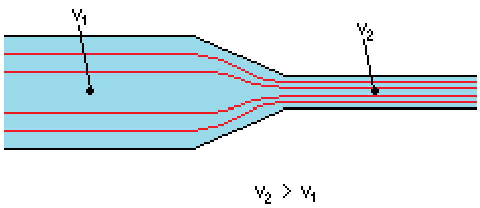
- Law of conservation of mass for fluid dynamics: The mass flux is the same at any point in a tube of flow. That is, $\rho A v = constant$

- Volume flux (or volume flow rate) R - the small amount of volume passing through tube cross section A in a short time. Given by $R = Av$
  - From $R = \frac{\delta V}{\delta t} = \frac{A v \delta t}{\delta t} = Av$

- For incompressible fluids, R is also constant.

- Equations of continuity - a class of equations that come from the principle that, if there are no "sources" or "sinks" in a volume of flowing material, the mass within the volume is constant. Can also be applied to other "flowing" quantities such as electric charge.

- In diagrams representing flow, widely spaced streamlines indicate slow flow rate, and tightly spaced streamlines indicate fast flow rate at that point.



- Regions of higher fluid velocity have lower fluid pressure. Regions of lower fluid velocity have higher pressure.

3 Bernoulli's Equation

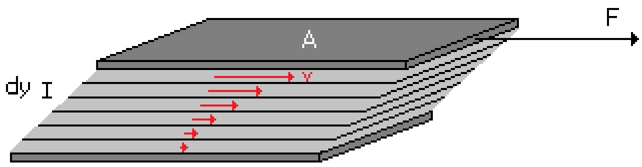
- Bernoulli's equation: In an ideal fluid, $p + \frac{1}{2}\rho v^2 + \rho g y = constant$ for any two points along a tube of flow.

- Comes from applying $\Delta K + \Delta U = W_{ext}$ to a small mass in a tube of flow, where the W_{ext} contributions are from gravity, fluid pressure at the initial point, and fluid pressure at the final point; then applying conservation of mass ($A_1 v_1 = A_2 v_2$)

- Static pressure - the pressure that would be present in the fluid even if it were not flowing. Given by $p + \rho g y$
- Dynamic pressure - pressure resulting only from the flow of the fluid. Given by $\frac{1}{2}\rho v^2$
- With these definitions in mind, we can think of Bernoulli's equation as: static pressure + dynamic pressure = constant
- Compressible fluids have an internal change in potential energy due to varying distances between molecules.
- Viscous fluids have an internal change in kinetic energy similar to frictional energy
- Dynamic lift - the force that acts on a body as a result of its motion through a fluid (for example, on an airplane wing)
- Boundary layer - a thin layer of fluid that tends to surround an object moving through a viscous fluid. Has very low velocity compared to the fluid around it

4 Viscosity and Turbulence

- Laminar flow - fluid flow in which speed varies layer-by-layer



- The force required to set up a laminar flow in a viscous fluid between two plates each of area A is given by $F = \eta A \frac{dv}{dy}$, where $\frac{dv}{dy}$ is the change in velocity across each "layer" dy (known as the "velocity gradient"), and η is the coefficient of viscosity of the fluid (units: $\frac{N \cdot s}{m^2}$)
- A fluid with higher η has more resistance to flow
- η depends not only on the material of the fluid, but also on its temperature (higher $T \rightarrow$ lower η)
- If the velocity gradient is constant for all layers, $F = \eta A \frac{v}{D}$, where D is the total plate spacing
- Poiseuielle's law: The mass flux of a viscous fluid flowing through a cylindrical pipe is given by $\frac{dm}{dt} = \frac{\rho \pi R^4 \Delta p}{8 \eta L}$ where ρ is the density of the fluid, R is the radius of the pipe, and Δp is the change in pressure over a length L of the pipe

- When laminar flow reaches high enough speeds, it transforms into turbulent flow.
  - Analogy: At low speeds, an applied force $\vec{F}$ must exceed the opposing frictional force $\vec{f}$ in order for a block to move across a surface. At high enough speeds, $\vec{F}$ might tip the block over. This is similar to a laminar flow transitioning into turbulent flow.
- Reynolds number - a dimensionless quantity that characterizes the flow of viscous fluids. For a pipe of diameter D, the Reynolds number is given by $R = \frac{\rho D v}{\eta}$. There is a "critical" value of R at which laminar flow transitions to turbulent flow, which tells us the critical velocity at which this transition occurs: $v_c = R\frac{\eta}{\rho D}$.
- This critical flow speed increases with viscosity.