Fluid Statics

Notes prepared by Megan L. Barry

Based on Physics, 5th Ed. by Resnick, Halliday, Krane (Ch. 15)

1 Fluids and Solids

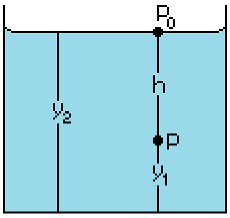
- Condensed matter - matter that is relatively incompressible, and whose densities stay constant as the temperature is varied. Includes liquids and solids.
  - Can usually be in equilibrium under applied compression, tensile, or shearing forces with minimal changes in shape
- Shearing force - a force applied parallel to the surface of an object
- Fluids - materials that flow easily under the action of a shearing force, and take the shape of their containers. Includes liquids and gases

2 Pressure and Density

- Pressure - a scalar quantity equal to the magnitude of the normal force per unit surface area. SI unit: 1 Pa (Pascal) = $\frac{N}{m^2}$
- A fluid under pressure exerts an outward force on any surface in contact with it.
- The force exerted by a fluid against a small area element is given by $\Delta\vec{F} = p\Delta\vec{A}$, where p is the pressure and $\Delta\vec{A}$ is a vector with magnitude equal to the small area and direction outward and perpendicular to the (closed) surface. Since $\Delta\vec{F} \parallel \Delta\vec{A}$, we can write $p = \frac{\Delta F}{\Delta A}$
- Density (ρ) - amount of mass per unit volume. A scalar quantity
  - For a small volume element Δv, $\rho = \frac{\Delta m}{\Delta V}$ where Δm is the mass of the volume element
  - For an object with constant density, $\rho = \frac{m}{V}$
- When the pressure on a material is increased by an amount Δp, its density increases, and its volume decreases by an amount ΔV
- Bulk modulus (B) - the ratio of the pressure increase to the fractional change in volume of a certain substance. That is, $B = -\frac{\Delta p}{\frac{\Delta V}{V}}$ (units: Pascals)
  - B is always positive (because Δp is positive and $\frac{\Delta V}{V}$ is negative)

3 Variation of Pressure in a Fluid at Rest

- If a fluid is in equilibrium, both the net force and net torque on every element of the fluid is zero.
- For a fluid in static equilibrium, the variation of pressure with depth is given by $\frac{dp}{dy} = -\rho g$, where y is the height above the reference level
- Weight density - weight per unit volume of a fluid, given by ρg
- The pressure difference between two depths is given by $\Delta p = -\int_{y_1}^{y_2} \rho g dy$, or $\Delta p = -\rho g \Delta y$ for a fluid with constant density (found by integrating $\frac{dp}{dy} = -\rho g$). Valid for ¡u¿any¡/u¿ container shape.
- For an open container, Δy is the distance from the surface (let's call it h), and $\Delta p = p_0 - p$ (where p_0 is 1 atm), so $p = p_0 + \rho gh$.
  - Thus, any point at a given height experiences the same pressure (for a uniform liquid)

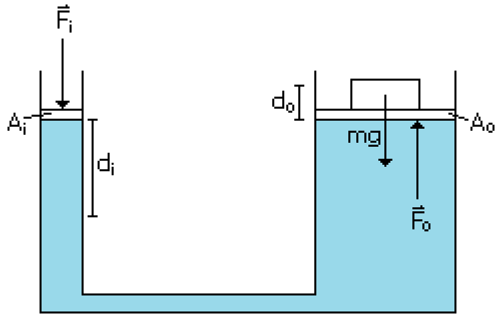


- By plugging in the proportionality relationship $\frac{\rho}{\rho_0} = \frac{p}{p_0}$ (where p_0 and ρ_0 are the atmospheric pressure and density at Earth's surface) into $\frac{dp}{dy} = -\rho g$ and integrating, we can find an expression for atmospheric pressure as a function of height: $p = p_0 e^{-\frac{h}{a}}$, where $a = \frac{p_0}{g\rho_0}$. The numerical value of a is the difference in altitude over which the pressure drops by e.

4 Pascal's Principle and Archimedes' Principle

- Pascal's Principle: If you increase the external pressure on a fluid at one location by an amount Δp, the same increase in pressure is experienced everywhere in the fluid. That is, $\Delta p = \Delta p_{ext}$.
- A hydraulic lever can be used to lift an object by applying a force much smaller than the object's weight.
  - The pressure p_i due to the input force F_i is equal to the pressure p_o due to the output force F_o, so $\frac{F_i}{A_i} = \frac{F_o}{A_o}$. To move the mass at constant velocity, $F_o = mg$, so $F_i = mg\frac{A_i}{A_o}$ ($A_i << A_o$, so F_i is small)
  - We can also find the distance d_o the mass moves through by using the fact that, for a compressible fluid, the displaced volumes will be

equal. $V = d_i A_i = d_o A_o$, so $d_o = \frac{d_i A_i}{A_o}$ ($A_i << A_o$, so d_o is small)

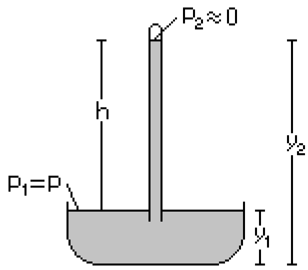


- Buoyant force ($\vec{F_b}$) - the net upward force on an object in a fluid due to the pressure difference between its top and bottom
- Archimedes' Principle: A body wholly or partially immersed in a fluid is buoyed up by a force equal in magnitude to the weight of the fluid displaced by the body.
- If an object of equal density to water is submerged in water, $\vec{F_b} = m\vec{g}$ and the object is in static equilibrium. If an object of higher density is submerged, $\vec{F_b} < m\vec{g}$ and the object sinks. If an object of lower density is submerged, $\vec{F_b} > m\vec{g}$ and the object floats until the submerged volume is equal to the volume of water that would have weight equal to $m\vec{g}$.
- If an object is placed on an underwater scale, the scale reads the net upward force on the object, $mg - F_b$
- Center of buoyancy - the center of gravity of the fluid displaced by the submerged part of a floating object (generally not the same as the object's center of mass)
- The buoyant force can be regarded as acting at the center of buoyancy.

5 Measurement of Pressure

- Gauge pressure - the difference between atmospheric pressure and the actual pressure of a fluid (i.e. the value a pressure gauge would display)
  - Positive gauge pressure $\rightarrow$ higher than atmospheric pressure
  - Negative gauge pressure $\rightarrow$ lower than atmospheric pressure
- Absolute pressure - atmospheric pressure plus gauge pressure (always positive)
- Let's apply $\Delta p = -\rho g \Delta y$ to a mercury barometer. $0 - p = -\rho g h \Rightarrow p = \rho g h$. That is, we can find the current air pressure by measuring the height

of the mercury.



6 Surface Tension

- Surface tension (γ) - the surface force F per unit length L over which it acts. That is, $\gamma = \frac{F}{L}$.
  - Multiplying by $\frac{\Delta x}{\Delta x}$ gives $\gamma = \frac{\Delta U}{\Delta A}$
- Surface tension can be thought of as extra resistance (per unit length) of a liquid's surface to an object attempting to pass through it.