Potential Energy

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Based on Physics, 5th Ed. by Resnick, Halliday, Krane (Ch. 12)

1 Conservative Forces

- Potential energy - energy that can be "stored" in a system
- Conservative force - a force which does zero work on a particle that moves around a closed path and returns to its starting point (ex: gravity, spring force). Always path-independent.
- Path-independent - describes a force for which the work done is the same no matter which path is taken between two points.

2 Potential Energy

- Potential energy is only defined for conservative forces.
- Potential energy is a property of a system, not an object.
- The change in potential energy due to an individual force is given by $\Delta U = -W$ where W is the work done by that force.
- Since it is ΔU we are usually interested in, not U_i or U_f, a reference point is usually chosen to have $U = 0$ (ex: the ground is oven to have $U_{gravitational} = 0$)
- Since $U(s) = -\int_0^x F_x(x)dx$, $F_x(x) = -\frac{dU(x)}{dx}$
- In a spring, the spring's relaxed state is usually chosen to have $U = 0$.
- Potential energy of a spring is given by $U(x) = \frac{1}{2}kx^2$ (from $U(y) = -\int_0^y -mgdy$)

3 Conservation of Mechanical Energy

- Since $\Delta K = W$ and $\Delta U = -W$, $\Delta K = -\Delta U$. As kinetic energy increases, potential energy decreases. (for isolated systems involving conservative forces)
- The total mechanical energy of a system involving only conservative forces is given by $E_{total} = K_{total} + U_{total}$

- Law of Conservation of Mechanical Energy: In an isolated system in which only conservative forces act, the total mechanical energy remains constant. That is, $\Delta E_{tot} = 0$, or $E_i = E_f$.

4 Energy Conservation in Rotational Motion

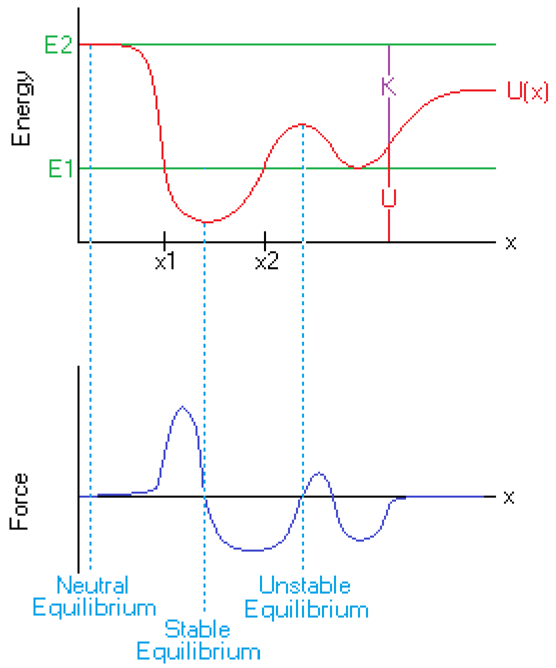
- The total kinetic energy of an object of mass M with instantaneous angular velocity ω rotating about an axis that passes through its center of mass is given by $K = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$

5 One-Dimensional Conservative Systems: The Complete Solution

- In one dimension, forces that depend only on position are always conservative.

- In 1D, solving $U(x)+\frac{1}{2}mv_x^2 = E_{tot}$ for velocity gives $v_x = \pm\sqrt{\frac{2}{m}[E - U(x)]}$. Thus, motion is only possible when $E > U(x)$.

- We can analyze the motion of a particle in 1D usbject to a force depending only on position by plotting $U(x)$ and comparing it to possible values of E_{tot}. (Remember: $F_x = -\frac{dU}{dx}$)



- The difference between E_{tot} and U on the graph gives K.

- A particle with energy E_1 can only move between points x_1 and x_2. A particle with energy E_2 is free to move anywhere.

- Stable equilibrium - describes the state of a particle at a local minimum of $U(x)$. F_x acts as a restoring force in the region surrounding this point.
- Unstable equilibrium - describes the state of a particle at a local minimum of $U(x)$. Any displacement of the particle from this point will result in it being pushed even farther away from the point.
- Neutral equilibrium - describes the state of a particle in a region of constant $U(x)$.
- Dissociation energy - the minimum energy required to break a molecule into separate atoms
- We can find $x(t)$ using energy instead of Newton's laws by integrating $\frac{dx}{dt} = \pm\sqrt{\frac{2}{m}[E - U(x)]}$ and solving for x. Seperate integrals must be carried out for positive and negative regions of v.

6 Three-dimensional Conservative Systems

- In three dimensions, $\Delta U = -\int_i^f \vec{F} \cdot d\vec{s}$ and $\vec{F}(x, y, z) = -\nabla U(x, y, z)$
  - The particle's path must be specified to solve the integral, but ΔU has the same value regardless of path.