Momentum

Notes prepared by Megan L. Barry

Based on Physics, 5th Ed. by Resnick, Halliday, Krane (Ch. 6)

1 Collisions

- Impulsive forces - forces that act for a very short time compared with the total time of observation of the system
- Any scenario in which two or more objects exert impulsive forces on each other can be treated as a collision, even if the objects don't touch (ex: a satellite that gravitationally slingshots around a planet, electric repulsion between atoms)

2 Linear Momentum

- Linear momentum $\vec{p} = m\vec{v}$
  - Like $\vec{v}$, $\vec{p}$ depends on the observer's reference frame
- $\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} => \sum \vec{F} = \frac{d\vec{p}}{dt}$ (another way to express Newton's second law)
  - Furthermore, $\vec{F}_{av} = \frac{\Delta\vec{p}}{\Delta t}$

3 Impulse and Momentum

- The impulse of a force is given by $\vec{J} = \int_{t_i}^{t_f} \vec{F}dt$
- Impulse-momentum theorem: The impulse of the *net* force acting on a particle during a time interval is equal to the change in momentum of the particle during that time interval. That is, $\vec{J}_{net} = \Delta\vec{p}$
- The relation $\Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt$ is true for all forces, not just impulsive forces.
- Impulsive forces tend to be much stronger than any external forces, so external forces such as gravity can be ignored when considering collisions.

4 Conservation of Momentum

- The total linear momentum of a system is given by $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$
- Since collisions are considered to have no external forces, $\sum \vec{F} = 0$, so $\frac{d\vec{P}}{dt} = 0$ ($\vec{P}$ is constant).
- Law of conservation of linear momentum: When the net external force acting on a system is zero, the total linear momentum of the system remains constant. That is, $\vec{P}_i = \vec{P}_f$ (for inertial reference frames)

5 Two-body Collisions

- In a two-body collision, $\Delta\vec{p}_1 = -\Delta\vec{p}_2$
  - Because $m_1\vec{v}_{1,i} + m_1\vec{v}_{2,i} = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f} => m_1(\vec{v}_{1,f} - \vec{v}_{1,i}) = -m_2(\vec{v}_{2,f} - \vec{v}_{2,i})$
- Center of mass frame - in collisions, the frame in which the initial and final momentum of the two-body system is zero
- Elastic - describes a collision in which the bodies bounce off one another, with their momenta unchanged in magnitude and reversed in direction in the CM frame
  - $v_{1,f} = -v_{1,i}$ and $v_{2,f} = -v_{2,i}$ in CM frame
  - Good approximation for rigid objects
- Inelastic - describes collisions in which the bodies rebound with reduced momenta in the CM frame
  - Good approximation for nonrigid objects
- Completely inelastic - describes collisions in which the bodies stick together, leaving them at rest in the CM frame
- Explosive - bodies rebound with increased momenta